MAX PLANCK INSTITUTE
FOR DYNAMICS OF COMPLEX
TECHNICAL SYSTEMS
MAGDEBURG

CSC
COMPUTATIONAL METHODS IN
SYSTEMS AND CONTROL THEORY

Reduced-order Modelling and Simulation of Gas Transportation Networks

Peter Benner
Joint work with Sara Grundel and Christian Himpe

Trends in Mathematical Modelling, Simulation and Optimisation: Theory and Applications

Virtual, 2–3 March 2021

Supported by:

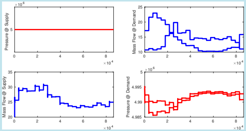
Federal Ministry
for Economic Affairs
and Energy

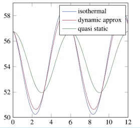
Math
Energy

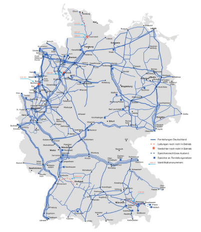
Simulation of German energy transportation networks

- **Goals:**
  - hierarchical modeling of transport and distribution networks
  - fast simulation on all levels
  - real-time scenario analysis for network operators
  - coupling of power and gas networks
- **Results:** New **discretization** and **model order reduction** methods for
  - isothermal Euler equations on network graph
  - with nonsmooth nonlinearity
  - leading to coupled system of differential-algebraic equations (DAEs)
  - with uncertain parameters

  Implemented in **morgen** — **M**odel **O**rder **R**eduction of **G**as and **E**nergy **N**etworks.







The German natural gas transportation network

Partners:

Fraunhofer SCAI
Fraunhofer ITWM
MPI Magdeburg
TU Berlin
HU Berlin
TU Dortmund
U Trier
PSI AG

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We will do some gas network ...

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- Modeling

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- Modeling
- Model simplification

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- Model discretization
- Model reduction
- Simulation experiments

Outline

1. Introduction

2. Modeling

3. Model Order Reduction

4. Outlook, Summary, Details

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- Regulatory requirements, real-time (15min decision horizon) control.
- Employ modern developments in numerics and reduced-order modeling.
- It remains a challenge!

Some gas network properties:

- $> 500,000$km gas pipelines in Germany[1] (earth-moon $< 400,000$km).

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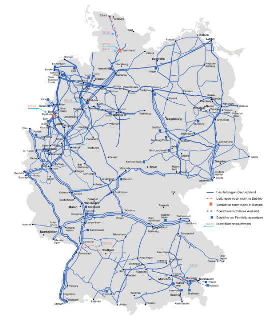
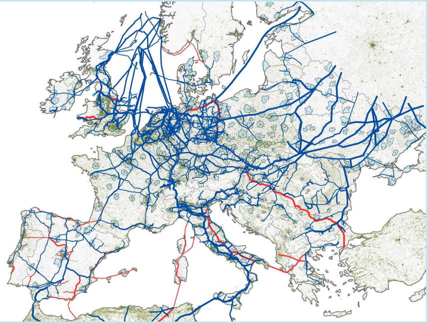
German gas transportation network **. . . embedded into European network.**





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- $> 500,000$km gas pipelines in Germany[1] (earth-moon $< 400,000$km).
- $> 240,000,000\text{m}^3$ natural gas consumed per day.[2].

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- Weather has effect on consumption **and** production.
- Planning horizon is 24h, operator decision horizon is 15min.

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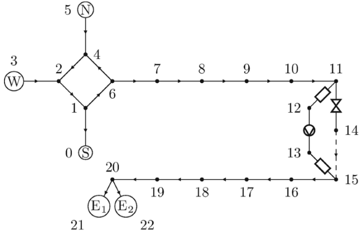
Modeling I: Pipeline

Friction-dominated isothermal Euler equations for 1D pipes:

$$
\begin{aligned}
\frac{1}{\gamma_0\, z_0}\, \partial_t p &= -\frac{1}{S}\, \partial_x q \\
\partial_t q &= -S\, \partial_x p - \Big(\underbrace{\frac{S\, g\, \partial_x h}{\gamma_0\, z_0} p}_{\text{Gravity}} + \underbrace{\frac{\gamma_0\, z_0\, \lambda_0}{2\, d\, S} \frac{q\, |q|}{p}}_{\text{Friction}} \Big)
\end{aligned}
$$

- Pressure: $p(x,t)$
- Mass-flux: $q(x,t)$
- Height: $h(x)$
- Temperature: T_0
- Diameter: d
- Cross-section: S
- Roughness: k
- Gas Const.: R_S
- Gas state: $\gamma_0(T_0, R_S)$
- Compress.: $z_0(T_0, p)$
- Friction: $\lambda_0(k, d)$
- Grav. accel.: g

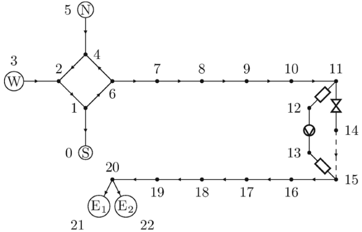


gas_N23_A24 from [Benner et al, 2019], modified from *GasLib-134*.

P. Benner, S. Grundel, C. Himpe, C. Huck, T. Streubel, C. Tischendorf (2019). Gas Network Benchmark Models. Applications of Differential-Algebraic Equations: Examples and Benchmarks, pp. 171-197, Springer, Cham.

Graph $(\mathcal{N}, \mathcal{E})$ incidence matrix $\mathcal{A}$:

$$\mathcal{A}_{ij} = \begin{cases} -1 & \mathcal{E}_j \text{ connects } \textbf{from } \mathcal{N}_i, \\ 0 & \mathcal{E}_j \text{ connects } \textbf{not } \mathcal{N}_i, \\ 1 & \mathcal{E}_j \text{ connects } \textbf{to } \mathcal{N}_i. \end{cases}$$



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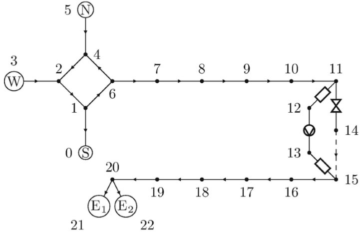

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Kirchhoff's laws:

1. The net mass-flux at every node is zero.
2. The sum of directed pressure drops in every loop is zero.

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Vectorized PDAE gas network model:

$$
\begin{aligned}
D_d\,\partial_t p^* &= D_q \partial_x q, \\
\partial_t q^* &= D_p \partial_x p - \left(D_g\, p^* + D_f\, \frac{q^*\,|q^*|}{p^*} \right), \\
\mathcal{A}_0\, q^* &= \mathcal{B}_d\, d_q, \\
\mathcal{A}_0^\intercal\, p^* &= \mathcal{B}_s\, s_p,
\end{aligned}
$$

- p^* is the pressure at a t.b.d. pipe location.
- q^* is the mass-flux at a t.b.d. pipe location.
- D_* are diagonal matrices.
- $\mathcal{A}_0$ is the incidence matrix without supply node rows.
- $\mathcal{B}_s$ is the incidence matrix of supply node rows.
- $\mathcal{B}_d$ is the incidence matrix of demand node columns.

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- Pipe midpoints:
  - (P)DAE tractability index bounded $\tau \leq 2$.
  - Given some weak topology constraints, PDAE becomes PDE [Grundel et al, 2014].
  - Boundary values affect friction term.

S. Grundel, L. Jansen, N. Hornung, T. Clees, C. Tischendorf, P. Benner (2014). Model order reduction of differential algebraic equations arising from the simulation of gas transport networks. In: Progress in Differential-Algebraic Equations, 183–205, Springer, Cham. doi:10.1007/978-3-662-44926-4_9.

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  - Boundary values affect friction term.
- Pipe endpoints:
  - (P)DAE tractability index bounded $\tau < 2$.
  - Given some weak topology constraints, PDAE becomes PDE.
  - Less oscillatory behaviour.

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- Averaged compressibility based on steady-state: $z(p, T, x, t) \to z_0$.

Simplification II: Model

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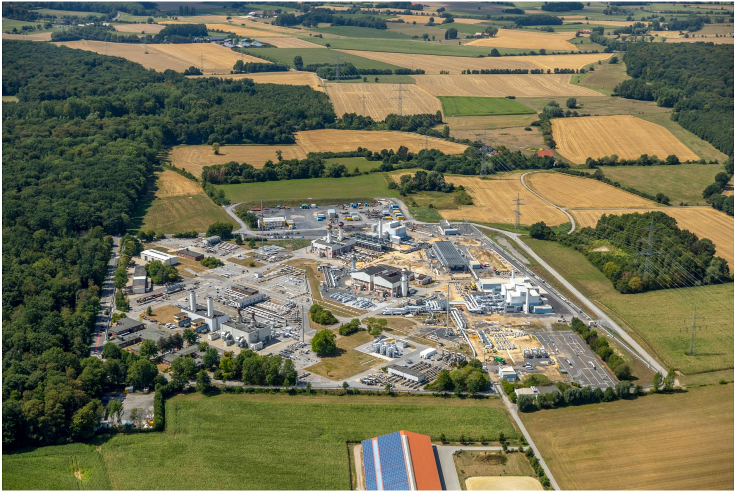
- Only step function boundary values.

Simplification III: Compressors



Natural gas compressor station in Werne, Germany, operated by Open Grid Europe.

Simplification III: Compressors

Simplified edge-based compressor models:

- Energy-based:

$$q_{\text{out}} = q_{\text{in}}$$

$$p_{\text{out}} = p_{\text{in}} \left(\frac{P_{\text{max}}}{p\gamma_0 z_0} \frac{\nu - 1}{\nu} + 1 \right)^{\frac{\nu}{\nu - 1}}$$

T.W.K. Mak, P. Van Hentenryck, A. Zlotnik, R. Bent (2019). Dynamic compressor optimization in natural gas pipeline systems. INFORMS Journal on Computing, 31(1):1–26. doi:10.1287/ijoc.2018.0821.

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- Affine*:

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Discretization I: Space

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- Implicit-Explicit (IMEX) methods are an appropriate tool.
- Consider: SSP optimality, stiff accuracy, passivity, efficiency.
- We recommend first order IMplicit-EXplicit method (i.e., combination of forward/backward Euler), providing often the best compromise between efficiency and accuracy, but other solvers are available in `morgen`, e.g. second-order IMEX (trapezoidal rule + SDIRK) with parametric Butcher tableau:

Explicit:

$$\begin{array}{c|cc} 0 & 0 & 0 \\ 1 & 1 & 0 \\ \hline & \frac{1}{2} & \frac{1}{2} \end{array}$$

Implicit:

$$\begin{array}{c|cc} \lambda & \lambda & 0 \\ 1-\lambda & 1-2\lambda & \lambda \\ \hline & \frac{1}{2} & \frac{1}{2} \end{array}$$

The Input-Output Model

Parametric, Structured, Nonlinear, Non-Normal, Square:

$$\begin{bmatrix} E_p(\theta) & 0 \\ 0 & I_{N_q} \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & A_{pq} \\ A_{qp} & 0 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} + \begin{bmatrix} 0 & B_{pd} \\ B_{qs} & 0 \end{bmatrix} \begin{bmatrix} s_p \\ d_q \end{bmatrix} + \begin{bmatrix} 0 \\ F_c + f_q(p, q, s_p, \theta) \end{bmatrix}$$

$$\begin{bmatrix} s_q \\ d_p \end{bmatrix} = \begin{bmatrix} 0 & C_{sq} \\ C_{dp} & 0 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix}$$

$$\begin{bmatrix} p_0 \\ q_0 \end{bmatrix} = \begin{bmatrix} \bar{p}(\bar{s}_p, \bar{d}_q) \\ \bar{q}(\bar{s}_p, \bar{d}_q) \end{bmatrix}$$

Input:

- Pressure at supply: s_p
- Mass-Flux at demand: d_q

State:

- Pressure: p
- Mass-Flux: q

Output:

- Mass-Flux at supply: s_q
- Pressure at demand: d_p

Two-step steady state algorithm:

1a. Linear mass-flux steady-state: $A_{pq}\,\bar{q} = -B_{pd}\,\bar{d}_q$

1b. Linear pressure steady-state: $A_{qp}\,\bar{p} = -\Big(B_{qs}\,\bar{s}_p + F_c\Big)$

2. Corrected pressure steady-state: $A_{qp} p_{k+1} = -\Big(B_{qs}\bar{s}_p + F_c + f_q(p_k, \bar{q}, \bar{s}_p, \theta)\Big)$

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- If more accuracy is needed, iterate with 1st order IMEX solver.

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Recap:

From: Hyperbolic 2D PDAE

To: Non-normal, coupled, nonlinear, parametric ODE

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- Perturbation system → Deviation from steady state
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- Coupled system → Structure-preserving methods

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- Large-scale → Low-rank computable methods*

Split reduction operators

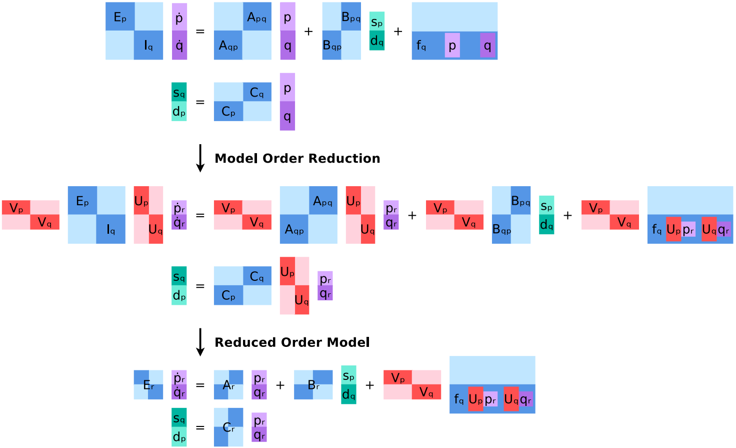
$$W = \begin{bmatrix} W_p \\ W_q \end{bmatrix} \in \mathbb{R}^{(N_p+N_q)\times r}$$

into structure-preserving reduction operator

$$\begin{bmatrix} W_p & \\ & W_q \end{bmatrix} \in \mathbb{R}^{(N_p+N_q)\times 2r},$$

where $W \in \{V, U\}$.

K. Kerns, A. Yang (1998). Preservation of passivity during RLC network reduction via split congruence transformations. IEEE Trans. CAD Integr. Circuits Syst. 17(7):582–591.



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Structured DMD Galerkin, via:	empirical reachability Gramian

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Spotlight: DMD-Galerkin

Plain Vanilla DMD:

$$X = [x_0 \;\; x_1 \;\; \dots \;\; x_T] \rightarrow \left\{ \begin{matrix} X_0 := [x_0 \;\; x_1 \;\; \dots \;\; x_{T-1}] \\ X_1 := [x_1 \;\; x_2 \;\; \dots \;\; x_T] \end{matrix} \right\} \rightarrow X_1 \overset{!}{\approx} \mathcal{A} X_0 \Rightarrow \mathcal{A} \approx X_1 X_0^+$$

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. . . with centering[4]

$$X \to \overline{X} := \big[(x_0 - \bar{x}) \quad (x_1 - \bar{x}) \quad \dots \quad (x_T - \bar{x})\big]$$

[4] S.M. Hirsh, K.D. Harris, J.N. Kutz, B.W. Brunton. Centering Data Improves the Dynamic Mode Decomposition. SIAM J. Appl. Dyn. Syst., 19(3): 1920–1955, 2020. doi:10.1137/19M1289881

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→ (Centered) DMD-Galerkin via (Discrete) Empirical Reachability Gramian!

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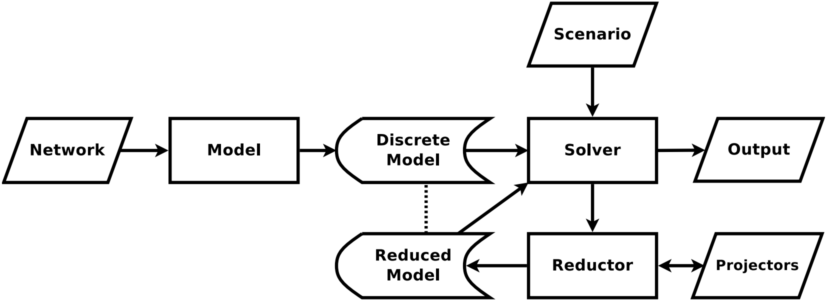
$\rightarrow$ No hyper-reduction implemented (yet).



Major modules:

- networks
- models
- solvers
- reductors
- tests

Minor modules:

- utils
- tools

Set-up

6

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- Short training, long testing

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  - `ebg_ro`, `ebg_wx`, `ebg_wz`
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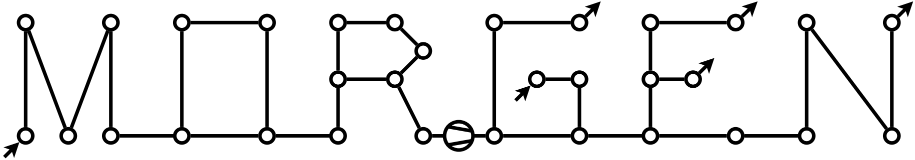
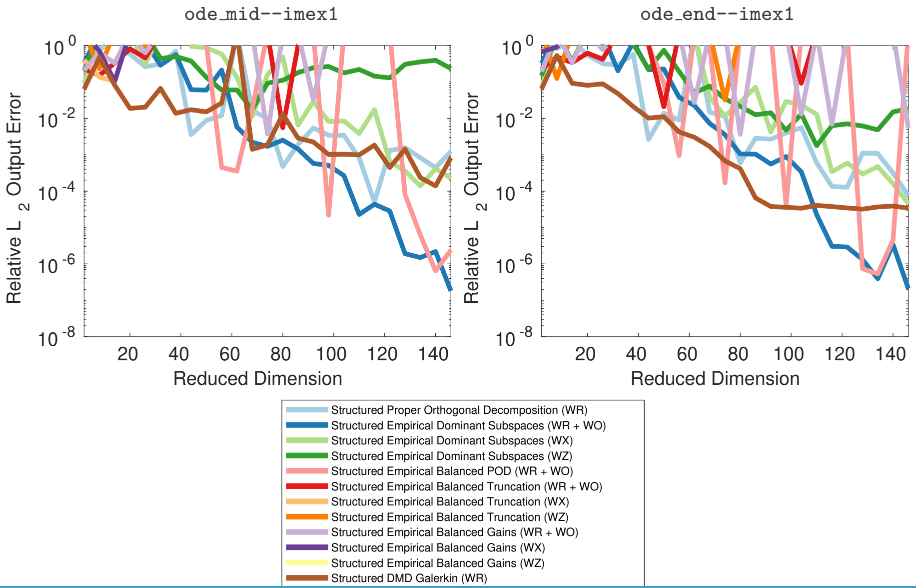
- Compare MORSCORE[6]

[6] C. Himpe (2020). Comparing (empirical-Gramian-based) model order reduction algorithms. arXiv [math.OC], arXiv:2002.12226.



- 2 Cycles
- 1 Compressor
- 2 Supply nodes
- 4 Demand nodes
- Pipe length $[20, 60]$km
- Time resolution 60s
- Temperature: $[0, 15]$°C
- Gas constant: $[500, 600]\frac{\text{J}}{\text{kg K}}$
- *Schifrinson* friction factor
- *AGA88* compressibility factor
- 900 States
- 6 Inputs & Outputs
- Training horizon: 1h
- Test horizon: 24h
- Perturbed steady-state training
- Standard load profiles testing

Experiment II: $L_2 \otimes L_2$ Model Reduction Error



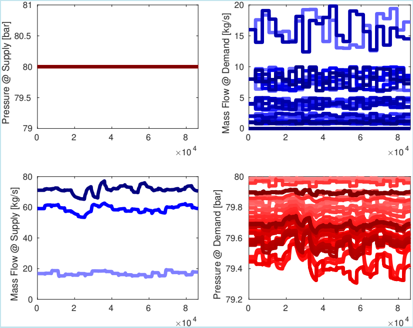
Experiment II: Evaluation

	ode_mid imex_1	ode_end imex_1	ode_mid imex_2	ode_end imex_2
pod_r	0.12	0.12	0.04	0.05
eds_ro	0.16	0.16	0.05	0.06
eds_wx	0.08	0.08	0.02	0.02
eds_wz	0.03	0.07	0.02	0.04
bpod_ro	0.07	0.07	0.02	0.02
ebt_ro	0.00	0.00	0.03	0.03
ebt_wx	0.00	0.00	0.00	0.00
ebt_wz	0.00	0.00	0.00	0.00
ebg_ro	0.00	0.01	0.02	0.02
ebg_wx	0.00	0.00	0.00	0.00
ebg_wz	0.00	0.00	0.00	0.00
dmd_r	0.14	0.18	0.03	0.04

MORscores $\mu(150, \epsilon_{\text{mach}(16)})$ in the $L_2 \otimes L_2$ norm for the "MORGEN" network.

The Network

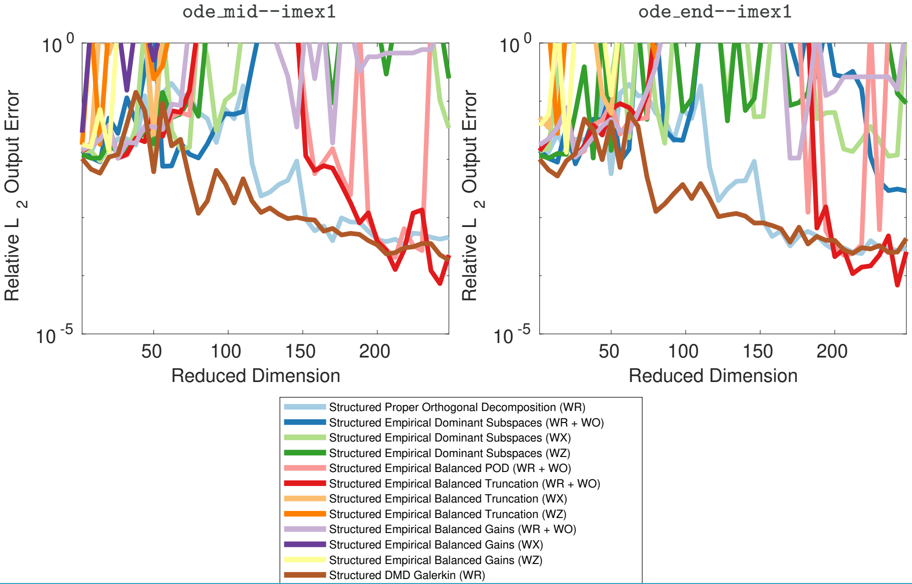
The Scenarios



- total length: 1412km
- 1 compressor

- steady-state, used as initial state:
  - pressure of 80bar at supply nodes and compressor;
  - demand mass-fluxes up to 16kg.

- 3886 states
- 48 inputs and outputs
- 20sec time steps


ode_mid--imex1
ode_end--imex1
10^0
10^-5
Relative L 2 Output Error
50
100
150
200
Reduced Dimension
Structured Proper Orthogonal Decomposition (WR)
Structured Empirical Dominant Subspaces (WR + WO)
Structured Empirical Dominant Subspaces (WX)
Structured Empirical Dominant Subspaces (WZ)
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Structured DMD Galerkin (WR)

1. Introduction

2. Modeling

3. Model Order Reduction

4. Outlook, Summary, Details

Some open problems and future work:

- Port-Hamiltonian model
- Parametric pipe roughness
- Intraday switchable valves
- Minimal training horizon
- SciGRID_gas network
- OGE partDE network

Conclusions from computational experiments:

- Prefer the endpoint model.
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The Software: `morgen` (Model Order Reduction for Gas and Energy Networks)

MATLAB code (Octave-compatible), under BSD 2-Clause License, available at:

`doi:10.5281/zenodo.4288510`